MONTHLY WEATHER REVIEW
VOLUME 96, NUMBER 6
JUNE 1968

# ON THE MOMENTUM EXCHANGE BETWEEN THE ATMOSPHERE AND EARTH OVER THE NORTHERN HEMISPHERE [1, 2]

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## ABSTRACT

This is an attempt to employ a pertinent boundary layer model and proper frictional parameters in the numerical analysis of the angular momentum budget of the atmosphere. The mean zonal surface stress due to friction is examined utilizing Lettau's boundary layer model and the earth's roughness study by the writer and Lettau for the period 1945–1955. The meridional and seasonal variations of the mean zonal surface stress are described; the mountain effect is derived indirectly; the stresses over the continent and ocean are compared; and finally the sensitivity of the numerical analysis to the prescribed frictional parameters is briefly discussed.

## 1. INTRODUCTION

Since Starr's [15] suggestion in 1948 on the importance of the absolute angular momentum transfer in the atmospheric general circulation, systematic efforts have been devoted to the study of the momentum budget (see Starr and Saltzman [16]). One of the basic concepts in the angular momentum study is that, considering the atmosphere and earth as a mechanically closed system, the atmosphere can receive or drain its angular momentum only through the exchange process with the earth. For a fixed volume of the atmosphere with its lower boundary at the earth's surface, assuming the local change to be negligible, this requires the net inflow of the angular momentum into the volume of the atmosphere to be balanced by the exchange with the earth.

The exchange of the angular momentum between the atmosphere and earth is provided by the surface torques due to frictional stress and mountain effect (see Starr [15], White [18], and Widger [19]). The analytical evaluation of the frictional stress has a special difficulty, because it involves one of the fundamental difficulties in the boundary layer theory, i.e., parameterization of the micrometeorological information for the use of the general circulation study. We recognize that presently there is no universally accepted boundary layer model for this purpose which completely incorporates all major turbulence characteristics and thermal effects. Nevertheless, with the progress in this field during recent years, it seems possible to apply the now existing boundary layer models in the study and to discuss the gross characteristics of the surface stress, provided we allow for the advantages and shortcomings of the particular model employed.

The writer, in one of his previous investigations (Kung [7]), used Lettau's [10] boundary layer model to estimate the kinetic energy dissipation in the boundary layer, and found very close agreement between the dissipation thus obtained and the dissipation in that layer obtained as the residual of the kinetic energy equation in the same study (also see Kung [8]). The latter dissipation value was obtained without employing specific theories and should be regarded as an independent check to the former. As the boundary layer dissipation is the scalar product of the surface frictional stress and the surface geostrophic wind, and the surface stress itself depends on the square of the surface geostrophic wind speed (see equation (4)), it is suggested that Lettau's model yields acceptable values of the surface stress and the deviation angle between the stress and isobar.

In this study Lettau's model was used with the regionally prescribed roughness parameter (see Kung [6] and Lettau and Kung [11]) and the 1000-mb. height data to describe the mean zonal surface stress for a period 1945–1955 over the Northern Hemisphere from 25° to 70°N. With Holopainen's [5] recently published momentum exchange data between the atmosphere and earth as estimated from the poleward flux of angular momentum, the mountain

[1] This research was supported by the Atmospheric Science Section, National Science Foundation, NSF Grant GA-1287.

[2] Contribution from the Missouri Agricultural Experiment Station Journal Series Number 5300.

effect is then estimated as the difference between total momentum exchange and the surface frictional stress. The surface stress over the continent and ocean are compared and discussed. The effects of the prescribed frictional parameters in the analytical study of the surface stress are also examined.

## 2. MEAN ZONAL SURFACE STRESS DUE TO FRICTION

The magnitude and direction of the daily surface geostrophic wind speed $V_g$ were computed for 360 points of a diamond grid system, determined by the intersections of longitudes and latitudes, from 25° to 70°N. with the 1000-mb. isobaric height data for an 11-yr. period from 1945 to 1955. For those grid points the aerodynamic roughness parameter $z_0$ was extracted over the continents from previous estimates (Kung [6] and Lettau and Kung [11]) for different seasons, and prescribed over the oceans for different $V_g$ values ($z_0 = 0.1$ cm. when $V_g \geqslant 10$ m./sec., and $z_0 = 0.01$ cm. when $V_g < 10$ m./sec.). The air density $\rho$ was calculated for the different months and latitudes with temperature data from Gentilli's [3] tabulation. The information thus obtained was utilized in conjunction with the following equations to compute the mean zonal surface stress due to friction $\bar{\tau}_{0x}$.

Regression equations may be established between the surface Rossby number $R_{o_s}$, the geostrophic drag coefficient $C$, and the deviation angle of the surface stress from the isobar $\alpha_0$ (in degrees) from Lettau's [10] analytical tabulations as unique valued functions of $R_{o_s}$;

$$C = 0.205/(\log_{10}R_{o_s} - 0.556) \pm 0.0004 \tag{1}$$

$$\alpha_0 = -3.03 + 173.58/\log_{10}R_{o_s} \pm 0.19 \tag{2}$$

where

$$R_{o_s} = V_g/(z_0 f) \tag{3}$$

and $f$ is the Coriolis parameter. Then the surface stress $\tau_0$ may be obtained according to Lettau's [10] formulation:

$$\tau_0 = \rho C^2 V_g^2. \tag{4}$$

With $\beta$ as the direction from which $V_g$ comes, $(\beta - \alpha_0)$ is used to obtain the zonal component of surface stress $\tau_{0x}$. The mean zonal surface stress $\bar{\tau}_{0x}$ is then obtained by taking the mean of $\tau_{0x}$ along the same latitudinal circle. The monthly average and annual mean of $\bar{\tau}_{0x}$ are listed at 5° intervals from 25° to 70°N. in table 1.

TABLE 1.—*Monthly and annual mean zonal surface stress $\bar{\tau}_{0x}$ over the Northern Hemisphere during 1945–1955 in units of dynes/m.*[2]

| Lat. | 70°N. | 65° | 60° | 55° | 50° | 45° | 40° | 35° | 30° | 25° |
|---|---|---|---|---|---|---|---|---|---|---|
| Jan | 0.10 | 0.22 | 0.32 | 0.64 | 1.01 | 1.00 | 0.63 | 0.60 | −0.00 | −0.51 |
| Feb | .09 | −.09 | −.02 | .46 | .94 | .93 | .59 | .49 | −.12 | −.66 |
| Mar | −.03 | −.17 | −.32 | .26 | .91 | .73 | .52 | .58 | −.09 | −.57 |
| Apr | −.19 | −.15 | −.11 | .28 | .68 | .55 | .30 | .26 | −.25 | −.43 |
| May | −.35 | −.17 | −.33 | −.01 | .35 | .54 | .22 | .18 | −.17 | −.24 |
| June | −.46 | .03 | .37 | .51 | .52 | .52 | .03 | −.00 | −.15 | −.14 |
| July | −.24 | −.12 | .52 | .56 | .56 | .41 | −.05 | −.15 | −.17 | −.10 |
| Aug | −.03 | .42 | .72 | .58 | .48 | .35 | −.12 | −.28 | −.28 | −.17 |
| Sept | −.02 | .02 | .56 | .93 | 1.10 | .59 | −.07 | −.32 | −.39 | −.31 |
| Oct | .25 | .36 | .82 | 1.35 | 1.57 | .88 | .25 | −.12 | −.44 | −.45 |
| Nov | .15 | .16 | .54 | 1.31 | 1.76 | 1.15 | .35 | .08 | −.54 | −.64 |
| Dec | −.00 | .04 | .34 | .81 | 1.70 | 1.14 | .57 | .33 | −.26 | −.73 |
| Annual mean | −.06 | .05 | .28 | .64 | .96 | .73 | .27 | .14 | −.24 | −.41 |

The meridional and seasonal variations of the mean zonal surface stress $\bar{\tau}_{0x}$ are apparent in table 1. The peak in the meridional profile of $\bar{\tau}_{0x}$ at the middle latitudes (45° to 55°N.) reaches the maximum of 1.7~1.8 dynes/cm.[2] in the winter and declines to 0.5 dyne/cm.[2] in the early summer. The peak of the negative $\bar{\tau}_{0x}$ at the lower latitudes reaches the maximum of −0.7 dyne/cm.[2] during winter and declines to −0.2 dyne/cm.[2] during summer. However, the negative $\bar{\tau}_{0x}$ in the higher latitudes seems to be in its maximum during the summer and in its minimum during the winter. The northward extension of the boundary between the lower latitude negative $\bar{\tau}_{0x}$ and middle latitude positive $\bar{\tau}_{0x}$ during the summer represents the northward extension of the surface easterly region during that season.

From the equations (1) through (4), the variations in the magnitude of $\bar{\tau}_{0x}$ should depend primarily on the zonal component of the surface geostrophic wind, since the large roughness length $z_0$ in the summer (see Kung [6]) is not enough to effectively increase the $\bar{\tau}_{0x}$ value through the corresponding increase of the geostrophic drag coefficient $C$.

## 3. FRICTIONAL STRESS AND MOUNTAIN EFFECT

With reasonable assumptions that the local change of the absolute angular momentum of a zonal ring of the atmosphere is negligible in dealing with the data for an extensive period to study the absolute angular momentum balance, and that the net meridional mass transport during the same period is negligible, the angular momentum equation states that the convergence of the poleward flux of the relative angular momentum is balanced by the angular momentum exchange between the earth and the atmosphere (see Starr and White [17]). Thus

$$\frac{\partial}{\partial y} R \cos\phi \int_0^H \oint \rho uv dx dz = -\int_0^H R\cos\phi\, \Delta p dz - \oint R\cos\phi\, \tau_{0x} dx \tag{5}$$

where $R$ is the radius of the earth, $\phi$ the latitude, $u$ the eastward wind component, $v$ the northward wind component, $\Delta p$ the pressure difference across mountain ranges, $H$ the highest peak of the mountain range at latitude $\phi$, and $x$, $y$, and $z$ the eastward, northward, and upward curvilinear distances. $R \cos\phi$ is supposed to represent sufficiently the distance from the earth's axis. Let us denote the zonal mean with the horizontal bar at the top, designate the poleward flux of the relative angular momentum as $F$

$$F=R\cos\phi\int_0^H \oint \rho uv dx dz, \tag{6}$$

and define the mountain effect in the momentum exchange $\overline{m}$ in parallel with $\overline{\tau}_{0x}$:

$$\overline{m}=\frac{1}{2\pi R\cos\phi}\int_0^H \Delta p dz. \tag{7}$$

Then from (5) we have the total linear zonal momentum exchange between the earth and atmosphere per unit area as a measure of the combined surface torques

$$-\frac{1}{2\pi R^2\cos^2\phi}\frac{\partial F}{\partial y}=\overline{\tau}_{0x}+\overline{m}. \tag{8}$$

Holopainen [5] recently evaluated the term on the left hand side of equation (8) over the Northern Hemisphere from 100 mb. to 1000 mb. from Crutcher's [2] upper wind statistics charts of the Northern Hemisphere. As the several years' period over which Crutcher's statistics were based is consistent with the period over which the 1000-mb. geostrophic wind speed was evaluated, and also since Holopainen's computation of the poleward flux of angular momentum over the Northern Hemisphere is consistent with widely accepted earlier studies (see Buch [1], Mintz [12], and Starr and Saltzman [16]), the difference between Holopainen's estimate of the left hand side of equation (8) and the estimate of $\overline{\tau}_{0x}$ in this study as listed in table 1 was ideally taken as $\overline{m}$.

Table 2 compares the evaluated $\overline{\tau}_{0x}$ and $\overline{m}$ from 25° to 70°N. for the multiannual basis. The listing in the table is in good qualitative agreement with previous studies for January 1946, by White [18] and Widger [19], and for November 1945, through February 1946, by Lorenz [12]. Namely, the mountain effect is a significant term in the earth atmosphere momentum exchange; the frictional surface stress and mountain effect $\overline{m}$ are of the same order of magnitude, and further in the middle latitudes, where the mountain effect is the most significant, it acts in the same direction as frictional stress to obstruct the eastward momentum from the atmosphere. There is also agreement with White's [18] study that the mountain effect is very small in the zonal ring from 50° to 70°N., and that it is significantly negative in the latitudes north of 60°N. The large middle latitude mountain effect tends to decrease toward the lower latitudes. However, due to the indirect method and assumptions used in deriving $\overline{m}$, only a qualitative discussion is appropriate on the multiannual basis.

## 4. FRICTIONAL STRESS OVER THE CONTINENT AND OCEAN

There have been many conjectures or speculations concerning the frictional stresses over the continental and oceanic areas since Priestley's [14] early study on the oceanic surface stress. With the boundary layer formulation and regionally prescribed roughness parameters as used in this study, it is possible to compare analytically the continental and oceanic surface stresses due to friction. Table 3 compares the winter (December–January) and summer (June–August) zonal surface stress $\overline{\tau}_{0x}$ over the North American Continent and North Atlantic Ocean. These are plotted on figures 1 and 2 separately for the winter and summer, with Hellerman's [4] updated estimate of the wind stress on the world ocean on the basis of multiannual compilation of wind roses.

It is readily observable that the magnitude of the stress value in the summer is comparable over North America and the North Atlantic, and that the stress value is even significantly larger over the North Atlantic than over North America during the winter in the middle

TABLE 2.—*Comparison of zonal momentum exchange due to surface stress $\overline{\tau}_{0x}$ and mountain effect $\overline{m}$ in units of dyne/cm.[2] and their ratio*

| Lat. | | 70°-60° N. | 60°-50° | 50°-40° | 40°-30° | 25° |
|---|---|---|---|---|---|---|
| Annual mean | $\overline{\tau}_{0x}$ | 0.09 | 0.65 | 0.66 | 0.06 | −0.42 |
| | $\overline{m}$ | −.27 | .05 | .39 | .31 | .02 |
| | $\overline{m}/\overline{\tau}_{0x}$ | −3.00 | .08 | .59 | 5.17 | −.05 |

TABLE 3.—*Winter and summer zonal surface stress $\overline{\tau}_{0x}$ over the North American Continent and North Atlantic Ocean*

| Lat. | 70°N. | 65° | 60° | 55° | 50° | 45° | 40° | 35° | 30° | 25° |
|---|---|---|---|---|---|---|---|---|---|---|
| Winter: | | | | | | | | | | |
| N. America | −0.10 | −0.37 | −1.29 | 0.69 | 1.06 | 1.90 | 0.72 | 0.15 | −0.62 | −0.99 |
| N. Atlantic | −1.17 | −.34 | 1.46 | 2.13 | 2.55 | 2.28 | 1.30 | 1.15 | .14 | −.72 |
| Summer: | | | | | | | | | | |
| N. America | −.13 | .18 | .71 | 1.08 | .92 | .77 | −.07 | .17 | .04 | −.04 |
| N. Atlantic | −.49 | −.28 | .44 | .94 | .97 | .93 | .33 | .09 | −.16 | −.41 |

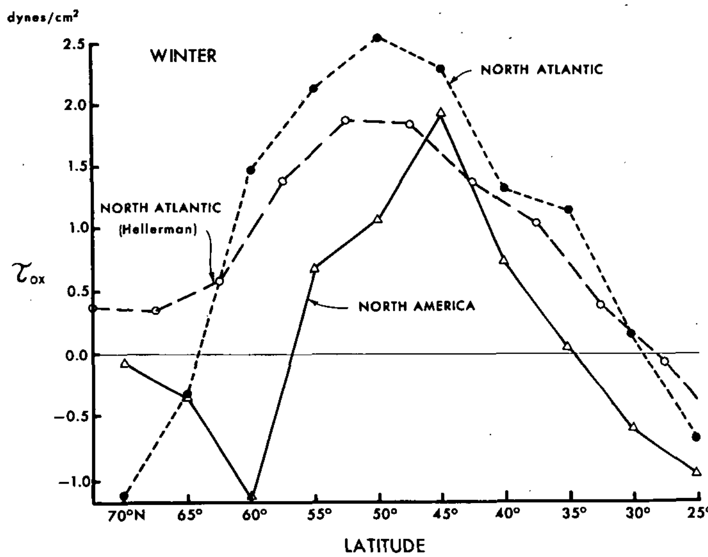


FIGURE 1.—Meridional profiles of the mean zonal surface stress over North America and North Atlantic in this study and in Hellerman's [4] study for winter.

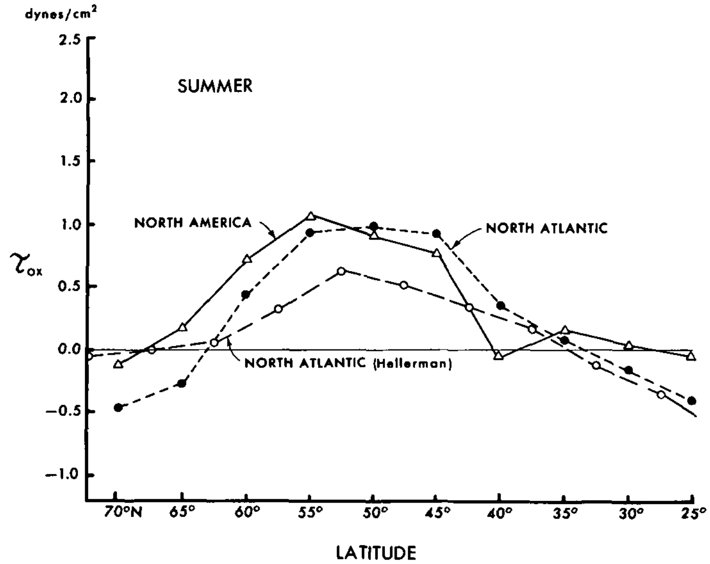


FIGURE 2.—Meridional profiles of the mean zonal surface stress over North America and North Atlantic in this study and in Hellerman's [4] study for summer.

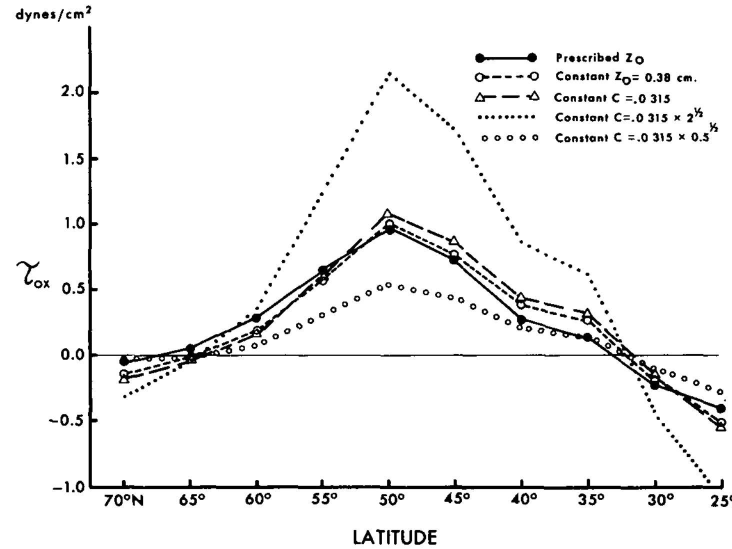


FIGURE 3.—Comparison of the meridional profiles of the mean zonal surface stress over the Northern Hemisphere obtained with various frictional parameters.

latitude. This is consistent with an existing argument that in the large-scale atmospheric circulation the effect of the large drag coefficient on the stress value is nearly cancelled by the small wind speed due to the large ground drag and vice versa. In one of his studies of energy dissipation, the writer (Kung [7]) pointed out that while the surface roughness parameter may be orders of magnitude smaller over the ocean than over the continent, the large geostrophic wind speeds over the ocean can well compensate the effect of the small roughness. The same argument applies here to interpret the stress values over North America and the North Atlantic; the seemingly large stress values over the North Atlantic is the reflection of the large surface geostrophic wind, which implies a strong surface wind, especially over the ocean.

The magnitude of Hellerman's [4] oceanic stress values for the same area of North Atlantic is consistently smaller than that of the stress values in this study. It is noted that while Hellerman's computation, as in the case of most stress studies, is based on the climatological data of actual wind observation, the computation in this study is based on the surface geostrophic wind. The surface geostrophic wind is an expression of the large-scale surface pressure pattern and has a certain practical advantage over the oceanic wind observations in dealing with the sparse data network.

Computation of the surface stress in this study employed Lettau's [10] barotropic boundary layer model. The baroclinic effects in the boundary layer were not incorporated. Lettau's [9] examination of the dependency of the geostrophic drag coefficient $C$ on the variation of the Richardson number at a 1-m. reference height shows that $C$ is smaller than the adiabatic values for surface cooling and larger under conditions of moderate surface heating. However, the computation based on the neutral condition should yield an acceptable stress value under normal conditions of the baroclinity in the boundary layer.

## 5. PRESCRIBED FRICTIONAL PARAMETERS

An interesting problem in the study of the large-scale atmospheric circulation is the sensitivity of the numerical model and the numerical analysis to the empirically prescribed frictional parameters. In addition to the stress computation with the regionally prescribed roughness parameter at the grid points, the stress value was also computed with various constant frictional parameters at all grid points, holding other data and parameters unchanged. The results illustrated in figure 3 includes, besides the standard case of regionally prescribed roughness parameter $z_0$, the annual means of the meridional zonal stress profiles from the cases of constant roughness $z_0=0.38$ cm. and constant geostrophic drag coefficients $C=0.0315$, $C=0.0315\times2^{1/4}$, and $C=0.0315\times0.5^{1/4}$. The constant $z_0=0.38$ cm. is the annual mean of the effective hemispherical mean roughness parameter, which is computed using the logarithm of the regionally prescribed values. The constant $C=0.0315$ is the annual mean of the hemispherical mean value of $C$ computed according to equation (1).

As shown in figure 3, there is no drastic deviation of the meridional profiles by constant parameters of $z_0=0.38$ cm. and $C=0.0315$ from the standard case. Although the regionally prescribed $z_0$ values vary over several powers of 10, especially over the continent, the employment of the mean $z_0$ or $C$ value in the analytical study still can give an acceptable result in the gross discussion of the large-scale characteristics of the atmospheric circulation as long as the prescribed constant value possesses a reasonable magnitude.

As the geostrophic wind data and other parameters cannot be altered in the numerical analysis, the obtained meridional profile of $\bar{\tau}_{0x}$ will vary in proportion to the magnitude of the prescribed constant $C^2$ as implied in equation (4) and shown in figure 3. However, the acceptable range of those constant parameters will be considerably larger in the numerical experiment of the general circulation, since we expect a certain adjustment between the wind and the frictional parameters to take place.

Although it is likely that a reasonable constant may be employed as the frictional parameter in the numerical analysis and numerical experiments in the gross discussions of the large-scale problems, and that the acceptable range of this constant may be rather wide in the numerical experiments, this does not preclude the importance of the regional prescription of the frictional parameters in the detailed study other than the meridional profile.

## 6. REMARKS

This study is merely an attempt to utilize a pertinent boundary layer model and proper frictional parameters in the numerical analysis of the large-scale problems. The results obtained are, of course, restricted by the data and assumptions in the boundary layer model employed. Thus, no definite claim should be made against the numerical value of the stress presented in this study. However, it is expected that the distribution of the zonal surface stress is described qualitatively, and consequent discussions of this paper may contain some points of interest to be examined in the future.

### ACKNOWLEDGMENTS

Mr. D. McInnis kindly reviewed the original manuscript. The author is very grateful to Mrs. B. E. Applegate and Mrs. P. C. Brewer in preparing the manuscript.

### REFERENCES

1. H. S. Buch, "Hemispheric Wind Conditions During the Year 1950," *Final Report*, Part 2, Contract No. AH 19-122-153, General Circulation Project, Dept. of Meteorology, Massachusetts Institute of Technology, 1954, 126 pp.
2. H. L. Crutcher, *Upper Wind Statistics Charts of the Northern Hemisphere*, Office of the Chief of the U.S. Naval Operations, Asheville, N.C., 1959, 3 vol.
3. J. Gentilli, *A Geography of Climate*, University of Western Australia, 1952, 107 pp.
4. S. Hellerman, "An Updated Estimate of the Wind Stress on the World Ocean," *Monthly Weather Review*, vol. 95, No. 9, Sept. 1967, pp. 607–626 (see corrected tables in vol. 96, No. 1, Jan. 1968, pp. 62–74).
5. E. O. Holopainen, "On the Mean Meridional Circulation and the Flux of Angular Momentum Over the Northern Hemisphere," *Tellus*, vol. 19, No. 1, 1967, pp. 1–13.
6. E. C. Kung, "Climatology of Aerodynamic Roughness Parameter and Energy Dissipation in the Planetary Boundary Layer of the Northern Hemisphere," *Annual Report*, Contract DA-36-039-AMC-00878, Dept. of Meteorology, University of Wisconsin, 1963, pp. 37–96.
7. E. C. Kung, "Kinetic Energy Generation and Dissipation in the Large-Scale Atmospheric Circulation," *Monthly Weather Review*, vol. 94, No. 2, Feb. 1966, pp. 67–82.
8. E. C. Kung, "Large-Scale Balance of Kinetic Energy in the Atmosphere," *Monthly Weather Review*, vol. 94, No. 11, Nov. 1966, pp. 627–640.
9. H. H. Lettau, "Wind Profile, Surface Stress and Geostrophic Drag Coefficient in the Atmospheric Surface Layer," *Advances in Geophysics*, vol. 6, 1959, pp. 241–257.
10. H. H. Lettau, "Theoretical Wind Spirals in the Boundary Layer of a Barotropic Atmosphere," *Beiträge zur Physik der Atmosphäre*, Frankfurt, vol. 35, No. 3/4, 1962, pp. 195–212.
11. H. H. Lettau and E. C. Kung, "The Aerodynamic Roughness of the Earth's Surface," (paper in preparation).
12. E. N. Lorenz, "Computation of the Balance of Angular Momentum and the Poleward Transport of Heat," *Scientific Report*, No. 6, Contract No. AF 19(122)-53, General Circulation Project, Dept. of Meteorology, Massachusetts Institute of Technology, Aug. 1951, 38 pp.
13. Y. Mintz, "Final Computation of the Mean Geostrophic Poleward Flux of Angular Momentum and of Sensible Heat in the Winter and Summer of 1949," *Final Report*, Contract AF 19(122)-48, Dept. of Meteorology, University of California at Los Angeles, Article 5, Mar. 1955, 14 pp.
14. C. H. B. Priestley, "A Survey of the Stress Between the Ocean and Atmosphere," *Australian Journal of Scientific Research*, Ser. A, vol. 4, No. 3, Sept. 1951, pp. 315–328.
15. V. P. Starr, "An Essay on the General Circulation of the Earth's Atmosphere," *Journal of Meteorology*, vol. 5, No. 2, Apr. 1948, pp. 39–43.
16. V. P. Starr and B. Saltzman (Compilers), "Observational Studies of the Atmospheric General Circulation," *Scientific Report*, No. 2, Contract No. AF 19(628)-5816, Planetary Circulation Project, Dept. of Meteorology, Massachusetts Institute of Technology, 1966, 698 pp.
17. V. P. Starr and R. M. White, "Balance Requirements of the General Circulation," *Geophysical Research Papers*, No. 35, U.S. Air Force, Cambridge Research Center, Dec. 1954, 57 pp.
18. R. M. White, "The Role of Mountains in the Angular-Momentum Balance of the Atmosphere," *Journal of Meteorology*, vol. 6, No. 5, Oct. 1949, pp. 353–355.
19. W. K. Widger, "A Study of the Flow of Angular Momentum in the Atmosphere," *Journal of Meteorology*, vol. 6, No. 5, Oct. 1949, pp. 291–299.

*[Received November 27, 1967; revised January 2, 1968]*